Camera Processing With Chromatic Aberration

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Abstract—Since the refractive index of materials commonly used for lens depends on the wavelengths of light, practical camera optics fail to converge light to a single point on an image plane. Known as chromatic aberration, this phenomenon distorts image details by introducing magnification error, defocus blur, and color fringes. Though achromatic and apochromatic lens designs reduce chromatic aberration to a degree, they are complex and expensive and they do not offer a perfect correction. In this work, we propose a new post-capture processing scheme designed to overcome these problems computationally. Specifically, the proposed solution is comprised of “chromatic aberration-tolerant” demosaicking algorithm and post-demosaicking chromatic aberration correction. Experiments with simulated and real sensor data verify that the chromatic aberration is effectively corrected.

Index Terms—Chromatic aberration, demosaicking, sensors.

I. INTRODUCTION

A digital camera is comprised of optics that focus light to the image plane, image sensor that converts light intensity to digital values, and signal processor that reconstruct the observed image from the sensor values. Thanks to advances in color image processing, image restoration algorithms can reverse the course of various hardware imperfections and signal distortions, including noise [27], [34], [37], [48]–[50], color crosstalk [26], faulty pixels [25], [40], blur [12], [47], [58], [62], and dynamic range [17], [30], [56], [57], [67].

In this paper, we address the problem of chromatic aberration. The image formation in camera is achieved by directing the light rays originating from the same point of the scene to a single point on an image plane. The optics system responsible for focusing the light is a series of lenses that bend light using refractive properties of glass (or similar materials). The refractive index of glass, however, is sensitive to the wavelengths of light, and making it difficult to achieve perfect focus when the incoming light covers a broad spectrum. Common problems stemming from the above limitations in optics are the magnification error (lateral chromatic aberration) and defocus blur (axial chromatic aberration) that are wavelength dependent. They both result in the so-called “color fringe” artifacts which severely deteriorate edges, textures, and high contrast regions of images, as shown in Figures 8(a) and 9(a).

Achromatic and apochromatic lens systems reduce (but not eliminate) chromatic aberration by compensating for nonidealities of one lens with another [3], though cost and size implications are substantial. Some authors proposed alternative materials to design a lens with high Abbe values [35]. Alternatively, one may seek to remove chromatic aberration in post-capture processing [5], [13], [16], [39], [41]–[44], [52], [53], [55], [59], [61], [66]. With the exception of [59], [61], however, these methods ignore demosaicking—the process of reconstructing the full color image from the sensor data that sampled by color filter array—which clearly fail when chromatic aberration is severe (see Figures 8(b-d) and 9(b-d)).

In this paper, we propose new camera processing scheme aimed at correcting the chromatic aberration. First, we design a demosaicking method that tolerates chromatic aberration. Modern demosaicking methods take advantage of the cross-color correlation, which break down when the chromatic aberration is severe. The proposed demosaicking method is designed to work even when the cross-color correlation is weak or defunct. Second, we propose a new post-demosaicking method for correcting the chromatic aberration based on the cross-color correlation principles of demosaicking. Combined, the resultant method is a fast and effective processing for eliminating the color fringes caused by the chromatic aberration.

II. BACKGROUND AND RELATED WORK

A. Chromatic Aberration

Most conventional single-chip color image sensors today make use of color filter array (CFA), a spatial multiplexing of absorptive red, green, and blue filters placed over an array of pixel sensors [4], [32]. The recorded sensor values \( Y_n \) at pixel location \( n = (x, y) \) represent the following:

\[
Y_n = \int c_n(\lambda) \ell_n(\lambda) \lambda = C_n \int \frac{r(\lambda)}{g(\lambda)} \ell_n(\lambda) d\lambda, \tag{1}
\]

where \( c_n(\lambda) = C_n \left( r(\lambda), g(\lambda), b(\lambda) \right)^T \) is the quantum efficiency of CFA at each pixel location; \( C_n \) is the filter indicator function (\( C_n = (1, 0, 0) \) for red pixel, for example); \( \lambda \) denotes spectral wavelength; \( \ell_n(\lambda) \) is the spectrum of the
incoming light; and $Z_n$ is the “complete” tristimulus value representation of $\ell_n$ in the sensor color space.

Chromatic aberration occurs because the refractive index of glass is sensitive to the wavelengths of light. This causes the focus to shift, as illustrated in Figure 1. One possible outcome of chromatic aberration is the axial color, which refers to the phenomenon that the perfect focus is achieved for only a finite set of wavelengths. For the remainder of visible wavelengths, the longitudinally displaced foci cause defocus blur, as illustrated in Figure 1. For a more complex lens system, the shape of the point spread function depends on the wavelength and the pixel location, and it is difficult to derive an analytical form. The impact of the chromatic aberration on image quality depends on the sensor quantum efficiency and the illuminant spectrum:

$$Z_n = \int \left( \frac{\tilde{r}(\lambda)}{\tilde{g}(\lambda)} \right) \int h(m, \lambda) \ell(n - m, \lambda) dmd\lambda,$$

(2)

where $h(n, \lambda)$ is the point spread function of wavelength-dependent defocus blur. The axial chromatic aberration can be attenuated by reducing the aperture opening size, which has the effect of make the “circle of confusion” in the image plane smaller. However, this comes at the cost of reduced light sensitivity, which is not desirable for low light photography.

The other possible outcome of chromatic aberration is the pixel magnification (lateral chromatic aberration). More specifically, the point at which the chief ray (the ray that pass through the center of the aperture opening) intersects the image plane depends on the wavelength. As a result, the effective image magnification is different for each wavelength:

$$Z_n = \int \left( \frac{\tilde{r}(\lambda)}{\tilde{g}(\lambda)} \right) \ell(t(\lambda)n, \lambda) d\lambda,$$

(3)

where $t(\cdot)$ is the wavelength-dependent magnification factor (see Figure 1). Since the quantum efficiency functions $\tilde{r}(\lambda), \tilde{g}(\lambda), \tilde{b}(\lambda)$ are not narrowband, the integral in (3) will blend the shifted pixels $t(\lambda)n$ together, meaning sensors free of axial chromatic aberration are still subject to blurring caused by lateral chromatic aberration.

Unlike axial chromatic aberration, the lateral chromatic aberration cannot be corrected by reducing the size of the aperture opening. A standard technique to correct this is by pixel resampling (or warping) to “register” red, green, and blue pixels to the same magnification factor [5], [66]:

$$\tilde{Z}_{c:n} = Z_c \left( \frac{t_2}{t_e} n \right),$$

(4)

where $t_1, t_2, t_3$ are the presumed magnification factors of red, green, and blue channels, respectively. Despite the popularity of pixel resampling, it has many shortcomings. First, the presumed magnification factors $t_e$ must be known in order to execute pixel resampling. In the extant literature, they are estimated using black-and-white calibration targets [52], [53], which is only practical for the case that optics is fixed when the camera is deployed (highly inconvenient for DSLR cameras that can exchange lenses). Others have proposed ad-hoc alternatives by replacing the calibration targets by detected achromatic edges in a natural scene [16], [41], [42]. Second, recall that magnification factor $t(\lambda)$ is wavelength-dependent. Since the color matching functions $\tilde{r}(\lambda), \tilde{g}(\lambda), \tilde{b}(\lambda)$ in (1) are broad bands, no magnification factors $t_1, t_2, t_3$ in (4) satisfies the notion of “color channel” magnification precisely—that is,

$$\int \tilde{r}(\lambda) t_1(n, \lambda) d\lambda \neq \int \tilde{r}(\lambda) t_2(n, \lambda) d\lambda \neq \int \tilde{r}(\lambda) t_3(n, \lambda) d\lambda \text{ etc.}$$

(5)

(unless the illuminant itself is narrowband). Third, $Z_c(t_2/t_e)n$ in (4) requires an interpolation—an easy task if $Z_n$ is a continuous image, but a difficult problem for finite pixel resolution measurements. Furthermore, the residual errors stemming from interpolation is color dependent, since different magnification factor is applied to each channel. In general, interpolated images are smoother. Although the color fringe due to lateral chromatic aberration is reduced by interpolation, the color fringe is still visible due to the fact that red and blue channels cannot match the sharpness of the green channel. It is worth noting also that resampling does not address the problem of axial chromatic aberration.

By the intuition that degraded achromatic edges result in green or purple color fringes, some chromatic aberration correction techniques desaturate detected green and purple edges [13], [16], [43]. However, they fail for boundaries of colorful objects, small image features, and/or for textured regions, leaving behind clearly visible residual errors (see [13] for examples). Another approach combines image deblurring and demosaicking [59]. Though the results are convincing, the numerical cost minimization requires iterative methods that are prohibitively slow for real-time applications. Generic color artifact reduction techniques lack specific ways to address edges displaced by magnification errors and recovering edge/texture details attenuated by axial chromatic aberration [55].

In practice, modern lens design regiments take aim at reducing both lateral and axial chromatic aberration, though the complexity and the cost of these lens systems vary significantly [3], [35]. Unlike the RGB color matching functions (which exhibit a large overlap between red and green channels), typical red and green sensor quantum efficiency functions do not overlap as much. As a result, the differences in blur (axial chromatic aberration) or relative magnifications (lateral chromatic aberration) between red and green pixels in the sensor space are expected to be large. In the remainder of this article, we assume that the camera is configured to bring green channel into focus (at least, to the best of the hardware’s ability). The goal of the proposed methodology is to match the quality of the red and blue channels to the green. Our method does not improve the quality of the green channel further.

B. Color Image Model and Camera Processing Pipeline

By camera processing we refer to the series of processing steps in digital camera to recover an image of interest $I_n$ from the acquired sensor data $Y_n$. This section describes the mapping between $Z_n$ and $I_n$. The process of recovering $Z_n$ from sensor data $Y_n$ is called demosaicking or CFA interpolation—this step is detailed in Section III.

The quantum efficiencies of pixel sensors in a conventional image sensor involve the spectral sensitivity of the sensor
material and the translucency of the color filter. Though these are carefully calibrated to be metameric, they are usually not identical to the color matching functions of some canonical RGB color space (such as the linear sRGB). Known as color correction, the mapping of sensor RGB data \( Z_n \) to the canonical RGB \( X_n \) is accomplished by a multiplication by color transformation matrix \( A \in \mathbb{R}^{3 \times 3} \):

\[
X_n = AZ_n.
\]  

(6)

The incoming light \( \ell_n(\lambda) \) is proportional to the spectral reflectance of the object represented in the scene and the spectrum of the incident light source. The human visual system has the ability to perceive the “color” of the object surface reflectance regardless of the light source color—a property commonly referred to as color constancy. Since the recorded image is not invariant to the color of the illumination, camera must determine the color intrinsic to the objects in various illumination in a process known as white balance. Prior work established that linear mapping is sufficient for attaining color constancy when the illuminant and reflectance spectra satisfy certain joint constraints [14], [19], [65]:

\[
I_n = W X_n = W A Z_n,
\]  

(7)

where \( W \in \mathbb{R}^{3 \times 3} \) is the white balance matrix. In order for an object surface with color-neutral spectral reflectance to map to a neutral trichromatic value, the eigenvalues of \( W \) must be inversely proportional to the color of the light source in its eigenspace [14]. Research in determining the eigenbasis of \( W \) optimal for white balance [14], [19], [65] and estimating the unknown illuminant color from recorded sensor values is still ongoing today [6], [9], [10], [15], [20], [21], [45], [64].

C. Color Image Model and CFA Sampling

We parameterize the image \( I_n \) to predict the behavior of \( Y_n \). One key property of white-balanced images in linear sRGB space is the idea that spatially highpass components of red, green, and blue channels are similar [2], [23], [24], [32]. That is, color radiance map \( I_n = (R_n, G_n, B_n)^T \in \mathbb{R}^3 \) is separable into lowpass (superscript LP) and highpass (superscript HP) components in the following manner:

\[
\begin{pmatrix}
R_n \\
G_n \\
B_n
\end{pmatrix} = \begin{pmatrix}
R_n^{LP} \\
G_n^{LP} \\
B_n^{LP}
\end{pmatrix} + I_n^{HP} 1
\]  

(8)

Here, \( I_n^{HP} \in \mathbb{R} \) is the highpass shared by all RGB channels and \( 1 = (1, 1, 1)^T \) is a neutral light in linear sRGB space. From this perspective, the lowpass represents the underlying “baseline” that describes textures and edges; and highpass encodes the “deviation” from this baseline.

Consider the reverse camera processing map \( I_n \mapsto Y_n \):

\[
Y_n = C_n Z_n = C_n M^{-1} I_n.
\]  

(9)

Predicting \( Y_n \) based on the model of \( I_n \) in (8),

\[
Y_n = C_n M^{-1} I_n^{LP} + I_n^{HP} 1
\]  

(10)

where \( e = M^{-1} (1, 1, 1)^T \in \mathbb{R}^3 \) represents the relative exposure of the sensor color channels [30] (i.e. sensor response when presented with a neutral-color image patch). One can equalize the relative exposures of each color channels by renormalizing the observation \( Y_n \) by the map \( Y_n \mapsto Y_n' \):

\[
Y_n' = \frac{Y_n}{C_n e} = C_n M^{-1} I_n^{LP} + I_n^{HP} C_n e
\]  

or alternatively, \( Y_n' = C_n (\text{diag}(e))^{-1} Z_n \). This has the effect of making the spatially highpass components of sensor red, green, and blue channels similar—compare (11) to (8) [30].

Consider for the moment the Bayer CFA pattern shown in Figure 3 (though the type of subsequent analysis applies to any CFA pattern [4]). One may rewrite \( C_n \) as follows:

\[
C_n' = \begin{pmatrix}
1 & (-1)^{n_x} & (-1)^{n_y} \\
1 & 1/4 & 1/2 \\
1 & 1/4 & -1/2 & 1/4
\end{pmatrix}
\]  

(12)

where the elements of the nonsingular matrix \( N \in \mathbb{R}^{3 \times 3} \) are the Fourier coefficients of \( C_n \). Denoting by \( Z_n' = N \text{diag}(e)^{-1} Z_n \),

\[
Y_n' = C_n' N^{-1} \text{diag}(e)^{-1} Z_n = C_n' Z_n'
\]  

(14)

\[
= Z_1' + (-1)^{n_x} + (-1)^{n_y} Z_2' + (-1)^{n_x+n_y} Z_3'
\]

Owing to the invertibility of \( N \), the problem of demosaicking is equivalent to the recovery of \( Z_n' \) from \( Y_n' \). This representation in (14) has the corresponding Fourier representation:

\[
\hat{Y}'(\omega) = \hat{Z}_1'(\omega) + \hat{Z}_2'(\omega - (0,\pi)) + \hat{Z}_2'(\omega - (\pi,0)) + \hat{Z}_3'(\omega - (\pi,\pi)),
\]  

(15)

where \( \hat{::} \) denotes discrete space Fourier transform and \( \omega \in [-\pi, \pi]^2 \) is the Fourier index. Noting that \( Z_{2,n}' \) and \( Z_{3,n}' \) are modulated, one can interpret the task of demosaicking \( Y_n' \mapsto Z_n' \) as a type of demodulation [18], [32].

The key advantage of the representation in (14) and (15) is that the transformation \( Z_n \mapsto Z_n' \) is effective for decoupling the broadband and lowpass signals. To see why this is the case, further analysis on (13) yields the following:

\[
Z_n' = N \text{diag}(e)^{-1} M^{-1} I_n^{LP} + I_n^{HP} 1
\]  

(16)
In other words, the highpass content \( I_n^{HP} \) is only present in the baseband signal \( Z_{1,n} \). By contrast, the modulated signals \( Z_{2,n} \) and \( Z_{3,n} \) are bandlimited and typically represents chrominance component of the color image. As illustrated in Figure 4(a) and noted by [18], [32], the overlapping support of the summands in (15) indicate aliasing—e.g., both \( \hat{Z}_1'(\omega) \) and \( \hat{Z}_2'(\omega - (\pi/2)) \) (or \( \hat{Z}_2'(\omega - (\pi/4)) \) or \( \hat{Z}_3'(\omega - (\pi/4)) \)) are nonzero for some value of \( \omega \). Demosaicking performance improves when the regions of overlap is reduced by the bandlimitedness of \( \hat{Z}_2'(\omega) \) and \( \hat{Z}_3'(\omega) \) that enjoy rapid spectral decay [32].

Many modern demosaicking methods recover \( Z_n \) from \( Y_n \) by implicitly or explicitly exploiting the lowpass-highpass coupling of color channels via the decomposition \( Z_n' \). Frequency domain decoupling \( Y_n' \mapsto \hat{Y}' \) played a prominent role in the demosaicking methods of [2], [18]. Similar analysis in the filterbank and wavelet domain has resulted in the methods of [1], [22], [28]. The analysis also led to binning processing [36], high dynamic range imaging [30], color filter array designs [32], and color image display designs [31]. The complete analysis of \( I_n \mapsto Z_n \mapsto \hat{Y}' \) is the subject of [30].

### III. Chromatic Aberration-Tolerant Demosaicking

One challenge of designing a demosaicking method for cameras likely to be subject to chromatic aberration is the fact that the model in (16) stemming from (8) does not hold. It is no
longer valid to assume that $Z'_2,n$ and $Z'_3,n$ in the transformed image $Z'_n$ are bandlimited images when the camera is subject to chromatic aberration. Indeed, Figure 2 shows the result of $Z'_n$ simulated from hyperspectral images with and without chromatic aberration. One can see greater image details (edges and textures) in Figures 2(e-f), confirming our assertion.

The key challenge for designing "chromatic aberration-tolerant demosaicking" is to recover $Z'_n$ despite relaxing the bandlimitedness assumption about $Z'_2,n$ and $Z'_3,n$. Increased risk of aliasing stemming from chromatic aberration is evident in the increased Fourier support of the modulated chrominance images $Z'_2(\omega - (\pi/n))$ and $Z'_3(\omega - (\pi/n))$ shown in Figures 4(b-c) overlapping the baseline $Z_1(\omega)$. (Support of $Z'_3(\omega - (\pi/n))$ increases also but does not pose risks of aliasing.) Axial chromatic aberration blurs the image, which reduces the Fourier support of $Z'_n$ slightly but not enough to counter aliasing. Indeed, demosaicking zipper artifacts are evidenced in regions of image dominated by chromatic aberration (see Figures 8(b-d) and 9(b-d)).

Below, we develop a demosaicking algorithm that relies less on the bandlimitedness and more instead on the notion of sparse representation to overcome potential hazards of aliasing. Drawing on the filterbank/wavelet-based demosaicking of [1], [22], [28], consider the Mallat wavelet packet transform shown in Figure 5(b) [51], [69]. Denote by $V_n^i$ and $U_n^i = (U_{1,n}^i, U_{2,n}^i, U_{3,n}^i)^T$ the transform coefficients of $Y_n^i$ and $Z_n^i$, respectively, where the subband index $i_j \in \{LL, LH, HL, HH\}$ in $i = (i_1, i_2, \ldots, i_j)$ indicates the separable orientation of the $j$th level decomposition. For example, $V_n^{LL}$ is the $LL$ wavelet coefficients of $Y_n^i$; $V_n^{LL,LL}$ is the $LL$ wavelet coefficients of $V_n^{LL}$; and $V_n^{LL,HH}$ is the $HH$ wavelet coefficients of $V_n^{LL,LL}$, etc.

Suppose we assume that the Mallat wavelet packet coefficients of $Z'_n$ are bandlimited in the following sense:

\[
U_{1,n}^{i_1} = 0 \quad \text{if } i_1 = HH \text{ and } J > \kappa \\
U_{2,n}^{i_1} = 0 \quad \text{if } i_1 \neq LL \\
U_{3,n}^{i_1} = 0 \quad \text{if } i_1 \neq LL \text{ or } J \leq \kappa,
\]

where $\kappa$ is the wavelet domain bandwidth, and $J$ is the total number of wavelet decompositions taken. Note that the bandwidth requirement in (17) is far less restrictive than the one typically used in traditional demosaicking. Specifically, the bandwidth of $U_{1,n}^i$ is comparable to $Z'_2(\omega)$ being supported in $[-\pi/2, \pi/2]^2$ in the Fourier domain—it is far larger than the bandwidth requirements of typical demosaicking techniques, as described by (15), and it suggests high likelihood of aliasing. To put this into the context of the present work, (17) is a more realistic model for handling the bandwidth of $U_{1,n}^i$ increased by chromatic aberration (as shown in Figures 4 and 2). One can also make use of model in (17) when images do not follow the behavior described in (8).

Based on the wavelet analysis by [33], we have the following relation between $U_{1,n}^i$ and $V_n^i$:

\[
\begin{align*}
V_n^{LL,i'} &= \frac{1}{4} \left( U_{1,n}^{LL,i'} + U_{2,n}^{LL,i'} + U_{2,n}^{HH,i'} + U_{3,n}^{HH,i'} \right) \\
V_n^{LL,\omega} &= \frac{1}{4} \left( U_{1,n}^{LL,\omega} + U_{2,n}^{LL,\omega} + U_{2,n}^{HH,\omega} + U_{3,n}^{HH,\omega} \right) \\
V_n^{HH,i'} &= \frac{1}{4} \left( U_{1,n}^{HH,i'} + U_{2,n}^{HH,i'} + U_{2,n}^{HH,i'} + U_{3,n}^{HH,i'} \right) \\
V_n^{HH,\omega} &= \frac{1}{4} \left( U_{1,n}^{HH,\omega} + U_{2,n}^{HH,\omega} + U_{2,n}^{HH,\omega} + U_{3,n}^{HH,\omega} \right) \\
\end{align*}
\]

(18)

where $i' = (i_2, i_3, \ldots, i_J)$. (Wavelet filters are slightly adjusted; see [33] for details.) Model in (17) simplifies (18) to

\[
V_n^i = \begin{cases} 
\frac{1}{4} U_{2,n}^{LL,i'} + \frac{1}{4} U_{1,n}^i & \text{if } i_1 \in \{LH, HL\} \\
\frac{1}{4} U_{3,n}^i & \text{if } i_1 = HH \text{ and } J > \kappa \\
\frac{1}{4} U_{1,n}^i & \text{else} 
\end{cases}
\]

The implied partitioning of $U_{1,n}^i$ and the subband-modulated $U_{2,n}^i$ and $U_{3,n}^i$ in $V_n$ are represented by yellow, cyan, and magenta in Figure 5, respectively. Since $V_n^{HH,\omega}$ and $V_n^{HL,\omega}$ share the same coefficient $U_{2,n}^{HH,\omega}$ (but $U_{1,n}^{HH,\omega} \neq U_{1,n}^{HL,\omega}$), we interpret this as an observation of $U_{2,n}^{HH,\omega}$ contaminated by $U_{1,n}^i$. The recovery of $U_{2,n}^{HH,\omega}$ from $V_n^{HH,\omega}$ and $V_n^{HL,\omega}$ is known as "robust regression" since the contaminant $U_{1,n}^i$ is sparse or heavy-tail distributed in the wavelet domain [22] (unlike typical image denoising-type contaminations). Our robust regression relies on the following principles:

**Theorem 1.** (Posterior Sparsity) Suppose $U_{1,n}^i$ is $\alpha$-sparse:

\[
P[U_{1,n}^i = 0] = \alpha.
\]

If the distribution of $U_{1,n}^i$ is symmetric about zero, then

\[
P[U_{1,n}^{HH,\omega} \neq 0 | |V_n^{HH,\omega}| > |V_n^{HL,\omega}|] \geq \alpha
\]

The proof of Theorem 1 appears in Section VII. Since $\alpha$ is typically close to 1 and the posterior probability that $U_{1,n}^{HH,\omega} = 0$ is even closer to 1 when $|V_n^{HH,\omega}| > |V_n^{HL,\omega}|$, the following scheme is almost always correct (i.e. perfect recovery with probability greater than $\alpha$):

\[
\begin{cases} 
(V_n^{HL,\omega}, U_{1,n}^{HH,\omega}) = \\
\{ (V_n^{HL,\omega}, V_n^{HL,\omega} - V_n^{HL,\omega}, 0), \text{ if } |V_n^{HH,\omega}| > |V_n^{HL,\omega}| \\
\{ (V_n^{HL,\omega}, 0, |V_n^{HL,\omega} - V_n^{HL,\omega}|) \text{ else} 
\end{cases}
\]

(20)
Intuitively, Theorem 1 compares $|V_{n}^{(LL^{'},i)}|$ and $|V_{n}^{(HL^{'},i)}|$ to decide which of the redundant copies of $U_{2,n}^{(LL^{'},i)}$ or $V_{n}^{(HL^{'},i)}$ is likely contaminated (by $U_{1,n}^{(HL^{'},i)}$ or $U_{1,n}^{(HL^{'},i)}$). The uncontaminated copy is used for reconstructing $U_{2,n}^{(LL^{'},i)}$.

To summarize, the chromatic aberration-tolerant posterior sparsity-directed demosaicking (PSDD) takes the form:

1. Compute equalized sensor data $Y_{n}$ from sensor data $Y_{n}$.
2. Compute Mallat wavelet packet coefficients $V_{n}^{i}$ from $Y_{n}$.
3. Assign $U_{1,n}^{(LL^{'})} = V_{n}^{(LL^{'},i)}$.
4. Assign $U_{1,n}^{(HL^{'})} = V_{n}^{(HL^{'},i)}$ for $J \leq \kappa$.
5. Assign $U_{1,n}^{(LL^{'},i)} = V_{n}^{(HL^{'},i)}$ for $J > \kappa$.
6. Assign $(U_{2,n}^{(LL^{'},i)}, U_{1,n}^{(HL^{'},i)})$ using (20).
7. Compute $Z_{n}$ from $U_{2,n}$ via inverse wavelet transform.
8. Recover $Z_{n}$ = $\text{diag}(e)N^{-1}Z_{n}$.

Many existing demosaicking methods select horizontal and/or vertical orientation for interpolation [18, 29]. These methods cannot handle the case when Fourier support of $Z_{n}$ is large (as in the case of chromatic aberration, where we assumed $[-\pi/2, \pi/2]^{2}$ above) because aliasing occurs in both horizontal and vertical directions. To avoid this, the orientation selection in PSDD occurs one wavelet subband at a time, giving a finer control over the orientation selection process. Some subbands are reconstructed by choosing horizontally oriented wavelet coefficients ($V_{n}^{(HL^{'},i)}$) while other subbands in the same image region may choose vertically oriented coefficients ($V_{n}^{(LL^{'},i)}$). Mallat wavelet packet transform of (18) has the effect of sparsifying the aliasing components $U_{1,n}^{(LL^{'},i)}$ and $U_{1,n}^{(HL^{'},i)}$, which guarantees PSDD recovery to be near-perfect demosaicking even with a large Fourier support of $Z_{n}$ (thanks to Theorem 1). The computational complexity of PSDD is roughly equal to the complexity of forward and inverse wavelet transforms, plus one comparator per wavelet coefficient to carry out (20). We implemented PSDD with overcomplete wavelet transform.

IV. CHROMATIC ABERRATION CORRECTION

Although experimental results will confirm that chromatic aberration-tolerant posterior sparsity-directed demosaicking (PSDD) successfully recovers complete sensor tristimulus value $Z_{n}$, this image still suffers from chromatic aberration, as in the case of (2) and/or (3). As noted earlier, existing solutions for correcting axial (reduced aperture size) and lateral (pixel resampling) chromatic aberrations have their limitations.

Before presenting the proposed method to eliminate the artifacts caused by chromatic aberration, we make general observations about magnification first. We claim that magnification affects highpass components more than the lowpass components. To see why, consider the error of representing sinusoid $e^{j\omega^{T}(t_{1} \cdot n)}$ by its magnified version $e^{j\omega^{T}(t_{1} \cdot n)}$:

$$
\| e^{j\omega^{T}(t_{1} \cdot n)} - e^{j\omega^{T}(t_{1} \cdot n)} \|^{2} = (e^{j\omega^{T}(t_{1} \cdot n)} - e^{j\omega^{T}(t_{1} \cdot n)})(e^{-j\omega^{T}(t_{1} \cdot n)} - e^{-j\omega^{T}(t_{1} \cdot n)})
= 2(1 - \cos(\omega^{T}(t_{1} \cdot (t_{1} - (t_{1} \cdot n)))).
$$

It is obvious that this error is small when the magnification factor $t_{1}$ is close to $t_{1}$, or in the regions of the image close to the center ($n \approx 0$). In addition, however, (21) also suggests that lowpass components ($\omega \approx 0$) are less penalized by the magnification compared to the highpass.

Recall that there are no magnification factor $t_{1}, t_{2}, t_{3}$ that corresponds exactly to the magnification of red, green, and blue channels in $Z_{n}$. Nevertheless, one conclusion we may draw from (21) is that the assumption

$$
\int \tilde{r}(\lambda) t(\lambda, n, \lambda) \, d\lambda \approx \int \tilde{r}(\lambda) t(\lambda, n, \lambda) \, d\lambda \quad \text{etc. (22)}
$$

holds for lowpass components of $t(\lambda, n, \lambda)$, even if $t_{1}$ is a non-exact approximation of $t(\lambda)$ over the ranges supported by $\tilde{r}(\lambda)$. Thus pixel resampling yields acceptable results for its lowpass components, while recovery of highpass details remains a challenge. As such, interpolation is justified for lowpass recovery as long as highpass is treated differently.

Based on the intuitions gained from (21) and the image model of (8), we propose a strategy for correcting chromatic aberration that relies on interpolation to recover lowpass components of the image and leverages redundancy across color components to reconstruct highpass components. Recall the color image model of (16). To determine the relative magnification factor $t_{2}/t_{1}$ and $t_{2}/t_{3}$ “good enough” for lowpass component recovery, we draw on the intuition that $Z_{n}$ and $\tilde{Z}_{3,n}$ are lowpass when the image is free of chromatic aberration. The following criteria is a sensible way to determine the best fit relative magnification factors:

$$
\max_{t_{1}, t_{3}} \| \tilde{Z}_{3,n} \| - \| \tilde{Z}_{2,n} \| - \| \tilde{Z}_{3,n} \|
\text{subject to } \tilde{Z}_{n} = N \text{diag}(e)^{-1}\tilde{Z}_{n},
$$

and $\tilde{Z}_{n} = (\tilde{Z}_{1,n}, \tilde{Z}_{2,n}, \tilde{Z}_{3,n})^{T}$ is as described in (4) for some interpolation method. Rudimentary interpolation methods such as bicubic interpolation suffice since we are only concerned here with the reconstruction quality of lowpass components.

On the other hand, the recovery of the highpass components is complicated by two factors. First, the correction of lateral chromatic aberration based on pixel resampling in interpolation of (4) is unsatisfactory since the interpolation process is imperfect (especially for highpass). The axial chromatic aberration also attenuates much of the highpass components from $\tilde{Z}_{1,n}$ and $\tilde{Z}_{3,n}$. Second, as already shown by (21), the non-exact approximation of $t(\lambda)$ by $t_{1}$ and $t_{3}$ causes errors in the highpass reconstructions, even with a perfect interpolation.

Thanks to the model in (8) and (10), however, we correct the interpolated image $\tilde{Z}_{n}$ by exploiting the redundancy of highpass components between the color channels. To restore the highpass components of $\tilde{Z}_{1,n}$ and $\tilde{Z}_{3,n}$, we propose a post-demosaicking process we call highpass replication (HPR):

$$
Z_{n}^{HPR} = h_{n} * Z_{n} + \frac{e}{e_{2}}(\delta_{n} - h_{n}) * Z_{2,n}
= M^{-1}I_{n}^{LP} + \frac{e}{e_{2}}(e_{1})^{HPR}
= M^{-1}I_{n}^{LP} + I_{n}^{HP} e,
$$

where $*$ denotes convolution, $h_{n}$ is a lowpass filter, and $\delta_{n}$ is an impulse function subject to $\delta_{n} - h_{n}$ is a highpass filter. Intuitively,
one may interpret (24) as overwriting the interpolated highpass of $Z_{1,n}$ and $Z_{3,n}$ with the more accurate highpass in $Z_{2,n}$.

V. EXPERIMENTAL RESULTS

We conducted both simulated and real data experiments. The synthetic experiments used hyperspectral image data of [11] as the original source image (77 images in total), which allows us to accurately model the spatial-spectral coupling in chromatic aberration before reducing to the RGB tristimulus values. The quantum efficiency of the color filters were modeled after an image sensor found in Nikon D300 [60]. We implemented a Gaussian optics model (singlet and achromatic lens), where the interactions with the hardware can be parameterized by only a few numbers. Our model also incorporates the modulation transfer function (MTF) to simulate a diffraction-limited lens. The lateral magnification was simulated with interpolation, and then the image was downsamped. Although this oversampling approach reduces the influence of the artifacts introduced by interpolation, it comes at the sacrifice of the spatial image resolution. This is justified because MTF relative to picture size goes up with the oversampling (reducing the effect of interpolation smoothing), and since the reduced resolution experiments are expected to be more difficult than what we expect from high density image sensors that is more typical in real world cameras.

The highpass replication was implemented with Gaussian lowpass filter $h_n$ with the standard deviation parameter $\sigma$. The filter parameter $\sigma$ was chosen to minimize the mean square error between the corrected image and the reference image in the sensor space ($Z_n$) using training hyperspectral image data. The optimization was performed using fminsearch in Matlab without demosaicking in the loop to prevent favoring any demosaicking algorithm or introduce additional experimental variable. The optimal $\sigma$ values were computed for two levels of chromatic aberration—we linearly interpolated between these $\sigma$ values to choose a new $\sigma$ parameter for a given chromatic aberration level. Separate optimization routines were used to train $\sigma$ values for singlet and achromat lens.

Peak signal to noise ratio (PSNR) and average $\Delta E$ error computed in S-CIELAB color space [70] were used to evaluate algorithm performance (averaged over 77 images in the hyperspectral image data set). Recalling that the goal of the demosaicking design (see Section III) is to tolerate chromatic aberration (i.e. recover chromatic aberrated full-color image from its CFA sampled version), demosaicking performance was evaluated with respect to reference image that is also chromatic aberrated. As shown in Figure 6 performance for the proposed chromatic aberration tolerant posterior sparsity-directed demosaicking (PSDD) method outperforms other state-of-the-art demosaicking algorithms [46], [54], [68] in both PSNR and S-CIELAB. The performance for the proposed PSDD method is largely invariant to the level of chromatic aberration (PSNR and S-CIELAB curves are relatively flat). By contrast, the performance of the alternative demosaicking methods LPA-ICI [54], LSLCD [46] and DLMMSSE [68] are very sensitive. It validates that demosaicking and correction of chromatic aberration can be handled separately if the demosaicking is made aberration-tolerant.

PSNR and S-CIELAB scores for the combination of demosaicking (Section III), pixel resampling/warping [5], and highpass replication (Section IV) are shown in Figure 7. While it is not surprising from earlier analysis that proposed PSDD yields superior demosaicking performance than the alternatives in presence of chromatic aberration, highpass replication is found to improve image quality in all cases, especially when the chromatic aberration is high (sometimes by more than 2dBs in PSNR). Unlike the trends evidenced in Figure 6, however, severe chromatic aberration will deteriorate image quality. Thus highpass replication can be seen as slowing (but not eliminating) the influence of chromatic aberration.

Figures 8 and 9 show the intermediary results (demosaicking only) on real image data taken by various consumer level cameras (specs given by Table I). We processed the raw sensor data using LPA-ICI LS LCD, DLMMSSE and the proposed PSDD demosaicking methods. The severity of zippering artifacts in the alternative algorithms increases with the chromatic aberration (see tree branches, petal patterns, and blue cast over purple yarn in Figure 8(b-d); purple cast over light blue yarn and resolution patterns in Figure 9(b-d)). This is particularly noticeable for LS LCD and DLMMSSE methods, which are designed to exploit the cross-color correlation. Besides zippering artifacts, the yarn and wedge patterns in Figure 9(b-d) shows the tendencies for existing demosaicking algorithms to slightly desaturate color fringing caused by chromatic aberration—likely because the recovered chrominance signals $Z_{2,n}$ and $Z_{3,n}$ are bandlimited (certainly untrue under the influence of chromatic aberration). Contrast this to the results of PSDD in Figures 8(a) and 9(a) where the of zippering artifacts are successfully prevented in regions of image with severe chromatic aberration. These observations support the assertion that sparsity is appropriate for demosaicking—even when the bandlimitedness are invalidated by chromatic aberration.

Figure 10 shows the results of full pipeline: demosaicking followed by pixel resampling (RS) and highpass replication (HPR). Pixel resampling in Figure 10(a) indeed reduces the effects of chromatic aberration, but it leaves behind residual errors near edges in textured regions. Possible causes include oversimplified modeling magnification factor (recall (5)) and imperfect interpolation, especially for high frequencies (recall (21)). Highpass replication in Figure 10(b) (our final result) successfully eliminates color fringes, leaving behind an image that is free of noticeable chromatic aberration distortions. As evidenced by Figures 10(c-d), however, zippering artifacts introduced by LPA-ICI demosaicking are still present after the resampling and highpass replication stages, and there is a noticeable loss of contrast for edges where chromatic aberration was present in the input (likely due to the fact that LPA-ICI never recovered the full bandwidth of the chrominance signals $Z_{2,n}$ and $Z_{3,n}$ that was increased by chromatic aberration). By contrast, the proposed PSDD preserved the contrast automatically.

Real data experiments in Figures 8-10 dispels confusions about the spatial resolution of digital cameras. It is becoming increasingly common for the image sensor resolution to exceed the optical resolution of the lens system, reducing the risks of aliasing in demosaicking. However, it is not enough to
overcome aliasing stemming from chromatic aberration in real camera systems, as evidenced by zippering artifacts in Figure 8(b-d). We conclude that cameras suffering from chromatic aberration still may be sensor resolution limited.

Finally, highpass replication has the effect of reducing random noise. This is likely due to the fact that highpass components of $Z_n$ are forced to be achromatic (reduces spatial variations of color) and consistent with the observations seen in [38] that have employed cross-color correlation of highpass for color image denoising. Though we do not imply here that highpass replication is a substitute for image denoising, the improvement is noticeable in real image data experiments.

VI. CONCLUSION

We proposed a new camera processing scheme aimed at correcting the chromatic aberration in two stages. First, we designed a chromatic aberration-tolerant posterior sparsity-directed demosaicking (PSDD) algorithm that does not require cross-color correlation. The interpolated image is provably near-perfect for images with sparse wavelet representation, and highpass replication is a substitute for image denoising, the improvement is noticeable in real image data experiments.

VII. PROOF OF THEOREM 1 (POSTERIOR SPARSITY)

Denote by $Q$ the event that $|V_n^{(LH,i)}| > |V_n^{(HL,i)}|$. By total probability, we have (25) (see page 12). Because $U_1^{(HL,i)} = U_1^{(LH,i)} = 0$ implies $V_n^{(LH,i)} = V_n^{(HL,i)}$, 

$$P\left[U_1^{(LH,i)} \neq 0 \mid Q, U_1^{(HL,i)} = 0 \right] = 1.$$ 

Hence it suffices to show that $P\left[U_1^{(HL,i)} = 0 \mid Q \right] > \alpha$ to prove the theorem. By Bayes, we have (26) (see page 12). By symmetry of distributions, we have $P\left[Q \mid U_1^{(HL,i)} \neq 0 \right] = \frac{1}{2}$. Furthermore, without loss of generality, assume that $U_2^{(LL,i)} > 0$ (same result applies when $U_2^{(LL,i)} < 0$). Then we have (27) (see page 12), where we used the fact that $P\left[U_1^{(LH,i)} > 0 \right] = 1/2$ by symmetry. Hence we
Fig. 8. Example intermediary results (demosaicking only; final results shown in Fig. 10(b)) for images taken with Camera #1. Raw sensor data processed by (a) PSDD (proposed), (b) LPA-ICI, (c) LSLCD, and (d) DLMMSE demosaicking methods. Zoom to see zippering artifacts and loss of chroma details. PSDD preserves edge structure without zippering while leaving chromatic aberration intact.
Fig. 9. Example intermediary results (demosaicking only; final results shown in Fig. 10(b)) for images taken with Camera #2 (left and middle) and Camera #3 (right). Raw sensor data processed by (a) PSDD (proposed), (b) LPA-ICI, (c) LSLCD, and (d) DLMMSE demosaicking methods. Zoom to see zippering artifacts and loss of chroma details. PSDD preserves edge structure without zippering while leaving chromatic aberration intact.
Fig. 10. Chromatic aberration correction applied to images in Figures 8 and 9. Raw sensor data is processed by (a) proposed PSDD demosaicking and pixel resampling (intermediary result) and (b) PSDD, pixel resampling, and highpass replication (final result). The results of (c) LPA-ICI demosaicking and pixel resampling, and (d) LPA-ICI, pixel resampling, and highpass replication are shown for comparison.
may conclude that the denominator of (26) is less than
\[ P\left| V_n^{(L,L)} \right| > \left| V_2^{(L,L)} \right| \] in the numerator, and it proves that
\[ P\left[ U_1^{(L,L)} = 0 \right] > \alpha. \]

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REFERENCES


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