Quantile Analysis Of Image Sensor Noise Distribution

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1. Goal Of This Work

- Sensor noise distribution does not match the noise model
- Denoising method not as effective with real sensor data.
- We compare sensor noise distribution to noise models using quantile analysis.

2. Experiment Setup

Data Acquisition
- Capturing X-Rite ColorChecker
- Detect non-uniformity of patches by ANOVA.
- Acquire another image with lens cap on.
- Nikon D90, Canon 550D, Fuji Pro1 in raw mode.
- ISO 1600, Shutter Speed 1/200 (1/250 Fuji), 1.1 Lux.

Heteroscedastic Pixel Noise Models

- Poisson Distribution: $k_g = \alpha \cdot g_0 + \beta$, $g_0 \sim \mathcal{P}(f_1)$ (1)
- Gaussian Distribution: $k_g \sim \mathcal{N}(\alpha f_2 + \beta, \alpha^2 f_3)$ (2)
- Poisson-Gaussian Hybrid Distribution: $k_g = \alpha \cdot g_0 + \beta$, $g_0 \sim \mathcal{P}(f_2)$, $g_0 \sim \mathcal{N}(\mu_g, \sigma_g^2)$ (3)

- $k$ is sensor observation, $g$ is affine transformed data and $f$ is latent intensity.
- $\alpha$ and $\beta$ are learned by regressing signal strength and variance.
- $f_1 = f_2 + \frac{\beta}{\alpha}$ and $f_3 = \frac{\sigma_g^2}{\alpha^2}$.
- $P_{\mu_g}$ is the sample mean of patches, and $P_{\mu_c}$ is the sample mean of capped image.

Quantile Analysis

Goal: Compare the distributions of sensor data and noise model

- $F_{data}$, $F_{model}$: $R \rightarrow [0, 1]$ are the cumulative distribution functions of data and model.
- QQ plot forms a $1:1$ line if data and model distributions are well-matched.

3. Pixel Noise

- Step 1: Choose two ColorChecker patches.
- Step 2*: Draw $h_0$ and $h_1$ at random.
- Step 3: Compute $w = h_0 - h_1$ and repeat Step 2 $\sim 3$.

4. Discrete Wavelet Transform (DWT) Noise

Haar DWT:
- $w = h_0 - h_1$

5. Discrete Cosine Transform (DCT) Noise

DCT: $d_k = \sum_{n=0}^{N-1} x(n) \cos \left( \frac{\pi}{N} (2n+1)k \right)$, $k \in \{0, 1, \ldots N-1\}$

- Step 1: Choose $X$ ColorChecker patches.
- Step 2*: Draw $(h_0, h_1, \ldots h_{X-1})$ at random.
- Step 3: Compute a DCT coefficient $d_k$ and repeat Step 2 $\sim 3$.

6. Multiplicative Multiscale Innovation (MMI) Noise

MMI: $\theta = (f_0 - f_1)/(2f_0 + f_2)$

- Step 1: Choose two ColorChecker patches.
- Step 2*: Draw $h_0$ and $h_1$ at random.
- Step 3: If $h_0 + h_1 \approx c$, keep $w = h_0 - h_1$. Repeat steps 2 $\sim 3$.

7. Noise Under Variance Stabilization

Goal: Map Poisson distribution to Gaussian distribution.

\[ g/(f) \sim \mathcal{P}(f) \rightarrow \eta(g) \sim \mathcal{N}(\eta(f), 1) \]

- Barlett/Anscombe: $\eta(k_f) = -2 \sqrt{(1/k_f - 1)/\mu + 1} 
\sim \mathcal{N}(\sqrt{T_f + \pi}, 1)$
- Generalized Anscombe: $\eta(k_f) = -2 \sqrt{(1/k_f - 1)/\mu + 1} \sim \mathcal{N}(2 \sqrt{T_f + \pi}, 1)$
- Haar Fisz: $\eta(k_f) = \frac{\mu_f}{\sqrt{T_f + \mu_f^2}} \sim \mathcal{N}(0, 1)$

8. Conclusions

- Real sensor noise distribution is heavier tailed than pixel/DWT noise models.
- Denoising method not as effective for real sensor data (more noisy).
- VS does not yield signal independent normal distribution of noise.
- Central limit theorem in effect for DCT: MMI noise model well-matched.